(b) An annuity on (x) provides k annually beginning at age $(x+n)$. Nothing is paid before n years. Annual premiums are payable for $m\ (<n)$ years beginning at x. Draw a cash flow diagram and obtain the expression for annual premium, assuming limiting age ω.

$\Rightarrow$ here,

- Premiums (P): Paid at the beginning of each year from age x to $x+m-1$ (total m payments)
- Annuity Benefits (k): Paid at the end of each year from age $x+n$ to the limiting age $\omega-1$ (or until death), but nothing before $x+n$.

CASH FLOW DIAGRAM :-

Time:	x	$x+1$	...	$x+m-1$	$x+m$	...	$x+n-1$	$x+n$	$x+n+1$	...	$\omega-1$
Premiums:	P	P	...	P	0	...	0	0	0	...	0
Benefits:	0	0	...	0	0	...	0	k	k	...	k

- Premiums, P are paid at times $0, 1, \ldots, m-1$ (relative to age x)
- Benefits are recieved at times $n, n+1, \ldots, \omega-x-1$

EPV of the deferred life annuity (benefit) at age $x+n$ is $\ddot{a}_{x+n}$ (annuity-due)

$\therefore$ Present value of benefit (EPV of deferred annuity)

$$EPV(\text{benefit}) = \ddot{a}_{x+n}$$

where,

$$\ddot{a}_{x+n} = \sum_{k=0}^{\omega-x-n} v^k \, {}_kP_{x+n}$$

is the value of whole life annuity due to starting at age $x+n$.

Discounting back to n years to age x:

$$EPV\text{ (benefit at age } x) = {}_nE_x \, \ddot{a}_{x+n}$$

where,

$${}_nE_x = v^n \, {}_nP_x$$

now,

$\therefore$ present value of premiums,

$$EPV\text{ (premiums)} = P \cdot \ddot{a}_{x:\overline{m}|}$$

where, $\ddot{a}_{x:\overline{m}|}$ is an m-year temporary life annuity due.

now, equating EPV,

$$P \cdot \ddot{a}_{x:\overline{m}|} = {}_nE_x \, \ddot{a}_{x+n}$$

So, the annual premium is,

$$P = \frac{{}_nE_x \, \ddot{a}_{x+n}}{\ddot{a}_{x:\overline{m}|}}$$

(c) A 60-year old couple purchases a joint life annuity that pays \$20,000 annually at the end of each year, ceasing at the second death. Write the EPV in terms of joint survival probabilities.

⇒ payments are made at the end of each year while at least one of the two is alive. That means,

It pays in year t if at least one of (60) and (60) is alive at time t. It stops after the second death.

This is a last-survivor annuity immediate.

Let,

x = male aged 60

y = female aged 60

${}_tP_x$ = Probability male survives t years

${}_tP_y$ = Probability female survives t years

${}_tP_{xy}$ = probability both survive t years

Probability that at least one is alive at time t:

$${}_tP_{xy}^{(1)} = {}_tP_x + {}_tP_y - {}_tP_{xy}$$

where, $P_{xy}^{(1)}$ denotes last survivor status.

For an annuity of 1 per year paid at end of year while at least one alive:

$$a_{xy} = \sum_{t=1}^{60-60} v^t \cdot {}_tP_{xy}^{(1)}$$

$$a_{xy} = \sum_{t=1}^{60-60} v^t \left[{}_tP_x + {}_tP_y - {}_tP_{xy}\right]$$

here, $v = \frac{1}{1+i}$ is the discount factor.

Now, applying to $20,000.

$$EPV = 20000 \cdot a_{xy}$$

$$EPV = 20000 \sum_{t=1}^{60-60} v^t [{}_tP_x + {}_tP_y - {}_tP_{xy}]$$

Since $x=60$, $y=60$, ~~whe~~ we have ${}_tP_x = {}_tP_y$ if same mortality table for both sexes, but they could differ problem says "a 60 year old couple" so often $x=y=60$ by P_x, P_y may differ if gender-specific mortality is used.

So, we can write,

$$EPV = 20000 \sum_{t=1}^{\omega-60} v^t ({}_tP_{60} + {}_tP_{60} - {}_tP_{60:60})$$