Split-Plot Design

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A design in which the plots of a set of treatments are split into several sub-plots to accommodate a second set of treatments is called a split-plot design.

This is a special kind of incomplete block design and is frequently used in factorial experiments. Here first set of treatments is called whole plot treatments or main plot treatments represented by factors A and second set of treatments is known as sub-plot treatments or split-plot treatments represented by factor B. The plots used for whole plot treatments are called whole plots or main plots while sub-plots are also called split-plots.

Example

Let there are three levels of irrigation [reserving, three different levels of water per plot and four dozes of Nitrogen fertilizer. We have to consider an area for studying the above treatment. If we divide the area into three whole plots and the whole plots are subdivided into four sub-plots, then the irrigation are allocated randomly into the whole plots and fertilizers are allocated randomly into sub-plot. If we repeat the process seceral time, then the resulting design is split plot design.

Advantages of SPD

i) An extra factor can be included in a SPD with little cost and this furnishes additional information quite cheaply and also increases the scope of experiment.
ii) More precise estimates of sub-plot treatments and its interaction with whole plot treatments are obtained in SPD than in RBD.
iii) Two or more factors needing relatively large and small units can be combined in the same experiment of SPD.
iv) The overall precision of SPD relative to RBD may be increased by arranging the whole plot treatments in an LSD or in an incomplete LSD.

Disadvantages of SPD

i) Main plot treatments are measured with less precision in SPD than in comparable RBD.
ii) When missing data occur, the increase in complexity of the analysis for the split-plot design is greater than for RBD.

When SPD is Needed?

Split-plot design is needed in the following situation:

i) In certain factorial experiments involving two factors. One factor A may require larger plot while other factor B may require smaller plot. Here SPD is preferable.
ii) SPD may used when an additional factor is introduced in an experiment to increase its scope.
iii) In some situations, one factor may be considered more important than another factor. So more information is desired on the important factor and on its interaction with the unimportant factor. In this case SPD may be applies with important factor as sub-plot and unimportant factor as whole plot treatment.
iv) When larger differences are guessed among the levels of certain factor than among the levels of others, then the levels of the factor having larger differences can be applied randomly to the whole plots in SPD.

Uses of SPD

Split-plot design derives its name from agricultural experimentations. This design is frequently used in agricultural, medical, and biological problem.

SPD is an incomplete block design:

In SPD, each whole plot may be regarded as block for subplot treatments but may also be treated as incomplete block as it contains only a fraction of the whole set of pq treatments. This is why; SPD may be looked upon as an incomplete block design.

SPD is a confounded design:

As an arrangement of the factors A and B with p and q levels respectively, in which the main effect A, with $(p-1)$ d.f, is completely confounded with incomplete block on whole plot differences. For this SPD is called a confounded design.

Comparison and Contrast of SPD with RBD

i) In a SPD with p whole plot treatments and q sub-plot treatments, the whole plot treatments are first randomly assigned to the whole plots and then sub-plot treatments are allocated randomly to the sub-plots within each whole plot. While in a RBD, pq treatment combinations are randomly assigned to the plots of a block.

ii) In SPD, main effect of sub-plot treatments and its interaction with main plot treatments are estimated and tested more precisely than the main effect of main plot treatments. But in a RBD, with factorial arrangement, the main effects and interaction are measured with equal precisions.

iii) A RBD occurs with one kind of plots and treatments while a two factor SPD occurs with two kinds of plots and treatments.

iv) RBD has one error component while SPD has two error components which are whole plot error and split-plot error.

v) Since RBD provides more d.f for error M.S than those for two error M.S in SPD, the test and interval estimates are more precise and efficient in RBD than in SPD.

vi) SPD can furnish additional information quite cheaply by introducing some extra factor with little extra cost. But RBD has no such scope.

vii) Analysis of data consists of two parts which are whole plot analysis and split-plot analysis in SPD.

viii) Whole plot error is usually larger than RBD error while split-plot error is less than RBD error.

Analysis of Split-Plot Design

Linear model

Let the model be

$$y_{ijl} = \mu + \alpha_i + \beta_j + \gamma_l + (\beta\gamma)_{jl} + e_{ijl} \quad ; \quad i = 1(1)r, \quad j = 1(1)p, \quad l = 1(1)q$$

where, y_{ijl} is the observation belonging to l^{th} sub-plot treatment on the j^{th} main plot in i^{th} replication, μ is the general mean effect, α_i effect due to i^{th} block (replication), β_j is the effect due to j^{th} main plot treatment of factor A, γ_l is the effect due to l^{th} sub-plot treatment of factor B, $(\beta\gamma)_{jl}$ is the interaction effect between j^{th} main plot treatment and l^{th} sub-plot treatment of factor A and B respectively, e_{ijl} is the random error component.

Assumption

Since each whole plot ay be regarded as block for sub-plot treatments, so sub-plot treatments are nor orthogonal. Thus the assumptions for the analysis of variance be

i) $E\left(e_{ijl}\right)=0$

ii) $E\left(e_{ijl}, e_{i'j'l'}\right)=\begin{cases}\sigma^2 & \text{if } i=i',\ j\neq j',\ l=l' \\ \rho\sigma^2 & \text{if } i=i',\ j\neq j',\ l\neq l' \\ 0 & \text{otherwise}\end{cases}$

iii) Observations are random

Restrictions: $\sum_i \alpha_i=\sum_j \beta_j=\sum_l \gamma_l=\sum_j (\beta\gamma)_{jl}=\sum_l (\beta\gamma)_{jl}=0$

Orthogonalization

Let us make the transformation

$$\begin{bmatrix}\mu_{ij}\\ z_{ij1}\\ z_{ij2}\\ \vdots\\ z_{ij\,q-1}\end{bmatrix}=\begin{bmatrix}\frac{1}{\sqrt{q}} & \frac{1}{\sqrt{q}} & \cdots & \frac{1}{\sqrt{q}}\\ a_{11} & a_{12} & \cdots & a_{1q}\\ a_{21} & a_{22} & \cdots & a_{2q}\\ \vdots & \vdots & \vdots & \vdots\\ a_{(q-1)1} & a_{(q-1)2} & \cdots & a_{(q-1)q}\end{bmatrix}\begin{bmatrix}y_{ij1}\\ y_{ij2}\\ y_{ij3}\\ \vdots\\ y_{ij\,q}\end{bmatrix}$$

Here, $\sum_{l=1}^{q} a_{l'l}=0$, $\sum_{l=1}^{q} a_{l'l}^2=1$, $\sum_{l=1}^{q} a_{l'l}a_{l''l}=0$; $l'\neq l''=1,2,\ \dots\ ,(q-1)$

$\sum_{l'=1}^{q-1} a_{l'l}^2=1-\frac{1}{q}$, $\sum_{l'=1}^{q-1} a_{l'l}a_{l'k}=-\frac{1}{q}$, $l\neq k=1,2,\ \dots\ ,q$

Now we get,

$$\mu_{ij}=\frac{1}{\sqrt{q}}\left(y_{ij1}+y_{ij2}+\ \dots\ +y_{ijq}\right)$$

$$=\frac{1}{\sqrt{q}}\left[\mu+\alpha_i+\beta_j+\gamma_1+(\beta\gamma)_{j1}+e_{ijl}+\ \dots\ +\mu+\alpha_i+\beta_j+\gamma_q+(\beta\gamma)_{jq}+e_{ijq}\right]$$

$$=\frac{1}{\sqrt{q}}\left[q\mu+q\alpha_i+q\beta_j+\sum\gamma_l+\sum(\beta\gamma)_{jl}+e_{ij.}\right] \qquad \text{since } \sum\gamma_l=\sum(\beta\gamma)_{jl}=0$$

$$=\sqrt{q}\left(\mu+\alpha_i+\beta_j\right)+\delta_{ij} \qquad \text{where } \delta_{ij}=\frac{e_{ij}}{\sqrt{q}}$$

Now it is observed that μ, α_i and β_j are to be estimated from μ_{ij}.

Again, $z_{ijl'}=a_{i'1}y_{ij1}+a_{l'2}y_{ij2}+\ \dots\ +a_{l'q}y_{ijq}$

$$=\sum_{l=1}^{q} a_{l'l}y_{ijl}$$

$$=\sum_{l=1}^{q} a_{l'l}\left(\mu+\alpha_i+\beta_j+\gamma_l+(\beta\gamma)_{jl}+e_{ijl}\right)$$

$$=\sum_{l=1}^{q} a_{l'l}\left(\gamma_l+(\beta\gamma)_{jl}+e_{ijl}\right)$$

$$=\sum_{l=1}^{q} a_{l'l}\left(\gamma_l+(\beta\gamma)_{jl}\right)+\sum_{l=1}^{q} a_{l'l}e_{ijl}$$

$$=\sum_{l=1}^{q} a_{l'l}\left(\gamma_l+(\beta\gamma)_{jl}\right)+\varepsilon_{ijl'} \qquad \text{where } \varepsilon_{ijl'}=\sum_{l=1}^{q} a_{l'l}e_{ijl}$$

So, γ_l and $(\beta\gamma)_{jl}$ are to be estimated from $z_{ijl'}$.

Now error of whole plot treatment is,

$$\delta_{ij} = u_{ij} - \sqrt{q}\left(\mu + \alpha_i + \beta_j\right)$$

and error of sub-plot treatment is $\varepsilon_{ijl'} = z_{ijl'} - \sum_{l=1}^{q} a_{l'l}\left[\gamma_l + \left(\beta\gamma\right)_{jl}\right]$

Here,
$$\begin{aligned} V\left(\delta_{ij}\right) &= v\left(u_{ij}\right) \\ &= v\left[\frac{1}{\sqrt{q}}\left[y_{ij1} + y_{ij2} + \ \dots \ + y_{ijq}\right]\right] \\ &= \frac{1}{q}\left[\sum v\left(y_{ijl}\right) + \sum_{l\neq}\sum_{l'} \operatorname{cov}\left(y_{ijl}, y_{ijl'}\right)\right] \\ &= \frac{1}{q}\left[q\sigma^2 + q\left(q-1\right)\rho\sigma^2\right] \\ &= \sigma^2\left[1 + \left(q-1\right)\rho\right] \end{aligned}$$

Again,
$$\begin{aligned} V\left(\varepsilon_{ijl'}\right) &= V\left(a_{l'1}y_{ij1} + a_{l'2}y_{ij2} + \ \dots \ + a_{l'q}y_{ijq}\right) \\ &= \sum_{l=1}^{q} a_{l'l}^2 V\left(y_{ijl}\right) + \sum_{l\neq}^{q}\sum_{k}^{q-1} a_{l'l}a_{l'k} \operatorname{cov}\left(y_{ij'l}, y_{ijk}\right) \\ &= \sigma^2 + \left(-\frac{1}{q}\right)q\rho\sigma^2 \\ &= \sigma^2\left(1-\rho\right) \end{aligned}$$

Let, $w_1 = \dfrac{1}{\sigma^2\left[1 + \left(q-1\right)\rho\right]}$ and $w_2 = \dfrac{1}{\sigma^2\left(1-\rho\right)}$

The weighted error sum of square is given by

$$\phi = w_1\sum_i\sum_j\left\{u_{ij} - \sqrt{q}\left(\hat{\mu} + \hat{\alpha}_i + \hat{\beta}_j\right)\right\}^2 + w_2\sum_i\sum_j\sum_{l'}\left\{z_{ijl'} - \sum_{l=1}^{q} a_{l'l}\left[\hat{\gamma}_l + \left(\hat{\beta}\gamma\right)_{jl}\right]\right\}^2$$

Now applying least square method, we obtain the estimate of the parameters as,

$$\hat{\mu} = \bar{y}_{...} \qquad \hat{\alpha}_i = \bar{y}_{i..} - \bar{y}_{...} \qquad \hat{\beta}_j = \bar{y}_{.j.} - \bar{y}_{...} \qquad \hat{\gamma}_l = \bar{y}_{..l} - \bar{y}_{...}$$

$$\left(\hat{\beta}\gamma\right)_{ij} = \bar{y}_{ijl} - y_{ij.} - \bar{y}_{..l} + \bar{y}_{...}$$

Partitioning of Total Sum of Squares

$$\begin{aligned} \sum_i^r\sum_j^p\sum_l^q\left(y_{ijl} - \bar{y}_{...}\right)^2 &= pq\sum_i\left(\bar{y}_{i..} - \bar{y}_{...}\right)^2 + qr\sum_j\left(\bar{y}_{.j.} - \bar{y}_{...}\right)^2 + pr\sum_l\left(\bar{y}_{..l} - \bar{y}_{...}\right)^2 + r\sum_j\sum_l\left(\bar{y}_{.jl} - \bar{y}_{.j.} - \bar{y}_{..l} + \bar{y}_{...}\right)^2 \\ &\quad + q\sum_i\sum_j\left(\bar{y}_{ij.} - \bar{y}_{i..} - \bar{y}_{.j.} + \bar{y}_{...}\right)^2 + \sum_i\sum_j\sum_l\left(y_{ijl} - \bar{y}_{.jl} - \bar{y}_{ij.} + \bar{y}_{.j.}\right)^2 \\ &= S_1 + S_2 + S_3 + S_4 + S_5 + S_6 \end{aligned}$$

where, $\sum_i^r\sum_j^p\sum_l^q\left(y_{ijl} - \bar{y}_{...}\right)^2$ is Total Sum of Square, $pq\sum_i\left(\bar{y}_{i..} - \bar{y}_{...}\right)^2$ is sum of square due to replication, $qr\sum_j\left(\bar{y}_{.j.} - \bar{y}_{...}\right)^2$ is sum of square due to main plot treatment A, $pr\sum_l\left(\bar{y}_{..l} - \bar{y}_{...}\right)^2$ is sum of square due to sub-plot treatment B, $r\sum_j\sum_l\left(\bar{y}_{.jl} - \bar{y}_{.j.} - \bar{y}_{..l} + \bar{y}_{...}\right)^2$ is sum of square due to interaction AB, $q\sum_i\sum_j\left(\bar{y}_{ij.} - \bar{y}_{.i.} - \bar{y}_{.j.} + \bar{y}_{...}\right)^2$ is sum of square due to error-1, $\sum_i\sum_j\sum_l\left(y_{ijl} - \bar{y}_{.jl} - \bar{y}_{ij.} + \bar{y}_{.j.}\right)^2$ is sum of square due to error-2.

ANOVA Table:

S.V	d.f	SS	MSS	$E(MSS)$	F
Replication	$r-1$	$S_1 = pq\sum_i \left(\bar{y}_{i..} - \bar{y}_{...}\right)^2$	$s_1 = \frac{S_1}{r-1}$	$\sigma^2\left[1+(q-1)\rho\right] + \frac{qp}{r-1}\sum\alpha_i^2$	$F_1 = \frac{s_1}{s_3}$
Main plot Treatment (A)	$p-1$	$S_2 = qr\sum_j \left(\bar{y}_{.j.} - \bar{y}_{...}\right)^2$	$s_2 = \frac{S_2}{p-1}$	$\sigma^2\left[1+(q-1)\rho\right] + \frac{qr}{p-1}\sum\beta_j^2$	$F_2 = \frac{s_2}{s_3}$
Error-1	$(p-1)(r-1)$	$S_3 = q\sum_i\sum_j \left(\bar{y}_{ij.} - \bar{y}_{.i.} - \bar{y}_{.j.} + \bar{y}_{...}\right)^2$	$s_3 = \frac{S_3}{(p-1)(r-1)}$	$\sigma^2\left[1+(q-1)\rho\right]$	
Sub-plot treatment (B)	$q-1$	$S_4 = pr\sum_l \left(\bar{y}_{..l} - \bar{y}_{...}\right)^2$	$s_4 = \frac{S_4}{q-1}$	$\sigma^2(1-\rho) + \frac{rp}{q-1}\sum\gamma_l^2$	$F_3 = \frac{s_4}{s_6}$
Interaction (AB)	$(p-1)(q-1)$	$S_5 = r\sum_j\sum_l \left(\bar{y}_{.jl} - \bar{y}_{.j.} - \bar{y}_{..l} + \bar{y}_{...}\right)^2$	$s_5 = \frac{S_5}{(p-1)(q-1)}$	$\sigma^2(1-\rho) + \frac{r}{(p-1)(q-1)}\sum\sum(\beta\gamma)_{jl}^2$	$F_4 = \frac{s_5}{s_6}$
Error-2	$p(r-1)(q-1)$	$S_6 = \sum_i\sum_j\sum_l \left(y_{ijl} - \bar{y}_{.jl} - \bar{y}_{ij.} + \bar{y}_{.j.}\right)^2$	$s_6 = \frac{S_6}{p(r-1)(q-1)}$	$\sigma^2(1-\rho)$	
Total	$pqr-1$				

Hypothesis Testing and Decision Rule

We have to test the following three hypothesis and make decision with them:

i) H_0 : The whole plot treatments are homogeneous

i.e. $H_0 : \beta_1 = \beta_2 = \cdots = \beta_p = 0$

H_1 : at least one of $\beta_j \neq 0$

The appropriate test statistic is $F_2 = \frac{s_2}{s_3} \sim F_{(p-1),(r-1)(p-1)}$

If $F_2 > F_{\alpha\%,(p-1),(r-1)(p-1)}$, then H_0 is rejected at $\alpha\%$ level of significance and we conclude that treatments differ significantly.

If the null hypothesis $H_0 : \beta_j = 0$ is rejected, then we take another null hypothesis

$$H_o : \beta_j = \beta_{j'} \quad ; \quad j \neq j' = 1, 2, \cdots, p$$
$$H_1 : \beta_j \neq \beta_{j'}$$

The appropriate test statistic be

$$t_1 = \frac{\left|\bar{y}_{.j.} - \bar{y}_{.j'.}\right|}{SE\left(\bar{y}_{.j.} - \bar{y}_{.j'.}\right)} = \frac{\left|\bar{y}_{.j.} - \bar{y}_{.j'.}\right|}{\sqrt{\frac{2s_3}{qr}}} \sim t_{(r-1)(p-1)}$$

In some case, $H_0 : \beta_j^i = \beta_{j'}^i \;;\; j \neq j' = 1,2, \cdots, p$

The appropriate test statistic be $F = \frac{r\left(\bar{y}_{.jl} - \bar{y}_{.j'l}\right)^2}{2s_t^2} \sim F_{1,(p-1)(r-1)}$ where $s_t^2 = \sum_i\sum_j \left(y_{ij} - \bar{y}_{.jl} - \bar{y}_{ij.} + \bar{y}_{..l}\right)^2$

ii) H_0 : The sub-plot treatments are homogeneous

i.e. $H_0 : \gamma_1 = \gamma_2 = \cdots = \gamma_q = 0$

H_1 : at least one of $\beta_l \neq 0$

The test statistic is

$$F_4 = \frac{s_4}{s_6} \sim F_{(q-1),\, p(r-1)(q-1)}$$

If $F_4 > F_{\alpha\%,(q-1),\, p(r-1)(q-1)}$, then H_0 is rejected at $\alpha\%$ level of significance.

If H_0 is rejected, then we take another null hypothesis $H_0 : \gamma_l = \gamma_{l'} \quad ; \quad l = l' = 1, 2, \cdots, q$

The test statistic is

$$t_2 = \frac{\left|\bar{y}_{..l} - \bar{y}_{..l'}\right|}{\sqrt{\dfrac{2s_6}{pr}}} \sim t_{p(r-1)(q-1)}$$

If $t_2 > t_{\alpha/2,\, p(r-1)(q-1)}$, then H_0 is rejected at $\alpha\%$ level of significance.

If $H_0 : \gamma_l = \gamma_{l'}$ is rejected, we take $H_0 : \gamma_l^j = \gamma_{l'}^j \quad ; \quad l \neq l' = 1, 2, \cdots, q$

The test statistic be

$$t_3 = \frac{\left|\bar{y}_{.jl} - \bar{y}_{.jl'}\right|}{\sqrt{\dfrac{2s_6}{r}}} \sim t_{p(r-1)(q-1)}$$

If $t_3 = t_{\alpha/2,\, p(r-1)(q-1)}$, then H_0 is rejected at $\alpha\%$ level of significance.

Again if $H_0 : \gamma_l^j = \gamma_{l'}^j$ is rejected, we take $H_0 : \gamma_l^j = \gamma_{l'}^{j'}$ and the test statistic be

$$F = \frac{r\left(\bar{y}_{.jl} - \bar{y}_{.j'l'}\right)^2}{2s_l^2} \sim F_{1,(p-1)(r-1)} \qquad \text{where} \quad s_l^2 = \sum_i \sum_j \left(y_{ijl} - \bar{y}_{.jl} - \bar{y}_{ij.} + \bar{y}_{..l}\right)^2$$

iii) There is no interaction effect,

i.e. $H_0 : (\beta\gamma)_{jl} = 0$

$H_1 : at\ least\ one\ of\ (\beta\gamma)_{jl} \neq 0$

The appropriate test statistic be,

$$F_5 = \frac{s_5}{s_6} \sim F_{(p-1)(q-1),\, p(r-1)(q-1)}$$

If $F_5 > F_{\alpha\%,(p-1)(q-1),\, p(r-1)(q-1)}$, then H_0 is rejected at $\alpha\%$ level of significance.

Efficiency of SPD (relative to RBD)

The analysis of variance table of SPD can be written in the following form:

S.V.	d.f.	SS	MSS
Replication	$r-1$	S_1	s_1
A	$p-1$	S_2	s_2
Error-1	$(p-1)(r-1)$	S_3	s_3
B	$q-1$	S_4	s_4
AB	$(p-1)(q-1)$	S_5	s_5
Error-2	$p(r-1)(q-1)$	S_6	s_6
Total	$pqr-1$		

Let us suppose that there is no effect of A, B and AB, in that situation the effect of A is appropriately, equal to error-1 and combined effect of B and AB are approximately equal to error-2. The reduced ANOVA table will be:

S.V.	d.f.	SS	MSS
Replication	$r-1$	$(r-1)s_1$	s_1
Error-1	$r(p-1)$	$r(p-1)s_3$	s_3
Error-2	$rp(q-1)$	$rp(q-1)s_6$	s_6
Total	$pq(r-1)$	$pq(r-1)$	

If we consider the replication as a block then total mea sum of square is

$$\frac{r(p-1)s_3+rp(q-1)s_6}{r(p-1)+rp(q-1)}=\frac{(p-1)s_3+p(q-1)s_6}{p-1+pq-p}$$
$$=\frac{(p-1)s_3+p(q-1)s_6}{pq-1}$$

Thus the efficiency is

$$E=\frac{1/s_6}{\dfrac{1}{\dfrac{(p-1)s_3+p(q-1)s_6}{(pq-1)}}}$$
$$=\frac{(p-1)s_3+p(q-1)s_6}{(pq-1)s_6}$$

Estimation of Missing Value

Let the observation of the l^{th} sub-plot treatment related to the j^{th} whole plot treatment in the i^{th} replication is missing.

Let the value of the observation is x . This x might be estimated in such a way that $SS\left(error\right)$ will be least. Sum of square of error-2 with missing value can be written as

$$SS\left(error-2\right)=\sum_{i'}\sum_{j'}\sum_{l'}y_{i'j'l'}^{2}+x^{2}-\frac{1}{q}\sum_{i'}\sum_{j'}y_{i'j'.}^{2}-\frac{1}{q}\left(y_{ij.}+x\right)^{2}-\frac{1}{r}\sum_{j'}\sum_{l'}y_{.j'l'}-\frac{1}{r}\left(y_{.jl}+x\right)^{2}$$
$$+\frac{1}{qr}\sum_{j}y_{.j.}^{2}+\frac{1}{qr}\left(y_{.j.}+x\right)^{2}\quad ;\quad i\neq i',\ j\neq j',\ l\neq l'$$

We have to estimate the value of x from this SS in such a way that $SS\left(error-2\right)$ will be minimum,

$$\therefore\qquad \frac{\delta\, SS\left(error-2\right)}{\delta x}=2x-2\frac{\left(y_{ij.}+x\right)}{q}-\frac{2\left(y_{.jl}+x\right)}{r}+\frac{2\left(y_{.j.}+x\right)}{qr}=0$$

$$\Rightarrow\qquad x-\frac{x}{q}-\frac{x}{r}+\frac{x}{qr}=\frac{y_{ij.}}{q}+\frac{y_{ijl}}{r}-\frac{y_{.j.}}{qr}$$

$$\Rightarrow\qquad x\left(\frac{qr-r-q+1}{qr}\right)=\frac{1}{qr}\left(r\,y_{ij.}+q\,y_{.jl}-y_{.j.}\right)$$

$$\therefore\qquad x=\frac{r\,y_{ij.}+q\,y_{.jl}-y_{.j.}}{\left(q-1\right)\left(r-1\right)}$$

After inserting the estimated value for missing observations, we perform the usual analysis of variance.

Reasons for Two Types of Errors in SPD

Since the data obtained from split-plot design are not orthogonal, so we have to make some orthogonal transformation to estimate the parameters involved in the model of this design. The parameters are estimated in two stages, so there are two types of error in split-plot design.