Question :1 Find the mean μ, for the data that follows the normal distribution where the known data are {1.5, 2, 8, 10, 9} with two missing items, here $n=7$ and $k=5$. Suppose the initial guess value, $\mu = [X_0 \bmod 3]$ where X_0 = your class roll. Consider two decimal places for detailed calculation.

Soln:- The initial guess value, $\mu_0 = [X_0 \bmod 3]$

$X_0 = 112$

$\mu_0 = [112 \bmod 3]$

$= 1$

The MLE estimate for the mean as,

$$\hat{\mu}_1 = \frac{1.5+2+8+10+9}{7} + \frac{1+1}{7}$$

$$= 4.36 + 0.285$$

$$= 4.64$$

$$\hat{\mu}_2 = 4.364 + \frac{4.64+4.64}{7} = 5.68$$

$$\hat{\mu}_3 = 4.36 + \frac{5.68+5.68}{7} = 5.98$$

$$\hat{\mu}_4 = 4.36 + \frac{5.98+5.98}{7} = 6.06$$

$$\hat{\mu}_5 = 4.36 + \frac{6.06+6.06}{7} = 6.091$$

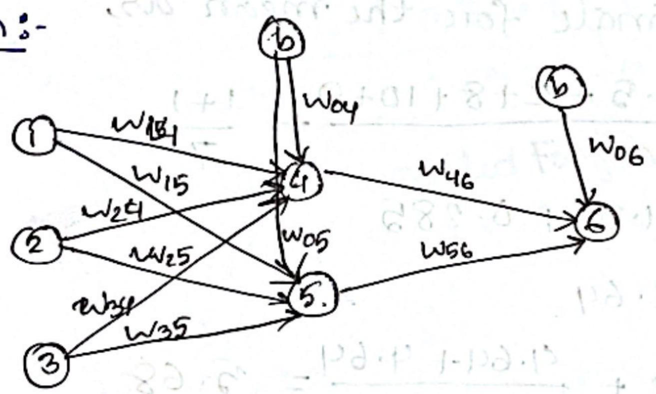
So, the mean is 6.091.

Question-2:- The inputs of ANN be $x_1=0.5$, $x_2=0.9$, $x_3=0.3$ at node 1, node 2, and node 3 respectively, the weights between input nodes to two nodes (node 4 and node 5) of the hidden layer are

$w_{14}^{(h)} = 0.3$, $w_{15}^{(h)} = 0.2$, $w_{24}^{(h)} = 0.5$, $w_{25}^{(h)} = 0.5$

$w_{34}^{(h)} = 0.7$, $w_{35}^{(h)} = 0.4$; and the weights between nodes of the hidden layer to the node (node 6) of the output layer are $w_{46}^{(o)} = 0.3$, $w_{56}^{(o)} = 0.5$. Also, the biases are $w_{04}^{(b)} = 0.3$, $w_{05}^{(b)} = 0.2$ at hidden layer, and $w_{06}^{(b)} = 0.4$ at output layer. The learning rate and target are 0.9 and 1 respectively. Find the updated weights after the first iteration. consider three decimal places for detailed calculation

Solution:-



i	x_i	$w_{i1}^{(h)}$ or $w_{i4}^{(h)}$	$w_{i2}^{(h)}$ or $w_{i5}^{(h)}$	$w_i^{(o)}$ or $w_{i6}^{(h)}$
1	0.5	0.3	0.2	0.3
2	0.9	0.5	0.5	0.5
3	0.3	0.7	0.4	
Bias →	1	0.3	0.2	0.4

The inputs $I_j = \sum_i w_{ij} O_i$

and outputs $O_j = \dfrac{1}{1+e^{-I_j}}$

Units i	Inputs, I_i	Outputs, O_j
4	$0.5\times0.3+0.9\times0.5+0.3\times0.7+1\times0.3=10.110$	$1/(1+\exp(-1.110))=0.752$
5	$0.5\times0.2+0.9\times0.5+0.3\times0.4+1\times0.2=0.870$	$1/(1+\exp(-0.870))=0.705$
6	$0.3\times0.752+0.5\times0.705+0.4\times1=0.978$	$1/(1+\exp(-0.978))=0.727$

Error at output layer for Unit j

$$Err_j = O_j(1-O_j)(T_j-O_j)$$

Error at hidden layer for unit j

$$Err_j = O_j(1-O_j)\sum_k Err_k w_{jk}$$

Unit i	Error Err_j
6	$0.727(1-0.727)(1-0.727)=0.054$
5	$0.705(1-0.705)(0.054)(0.5)=0.006$
4	$0.752(1-0.752)(0.054)(0.3)=0.003$

weights are updated $w_{ij}(new) = w_{ij}(old) + l\,Err_j\,O_i$

bias are updated by $\theta_{ij}(new) = \theta_{ij}(old) + l\,Err_j$.

weights, $w_{ij}(old)$	New weights, $w_{ij}(new)$
$w_{46}=$	$0.3+0.9\times0.752\times0.054=0.337$
$w_{56}=$	$0.5+0.9\times0.054\times0.705=0.534$
$w_{14}=$	$0.3+0.9\times0.003\times0.5=0.301$
$w_{15}=$	$0.2+0.9\times0.006\times0.05=0.203$
$w_{24}=$	$0.5+0.9\times0.003\times0.9=0.502$
$w_{25}=$	$0.5+0.9\times0.006\times0.9=0.505$
$w_{34}=$	$0.7+0.9\times0.003\times0.3=0.701$
$w_{35}=$	$0.4+0.9\times0.006\times0.3=0.402$
Bias	New bias
$\theta_{14}=$	$0.3+0.9\times0.003=0.303$
$\theta_{16}=$	$0.2+0.9\times0.006=0.205$
$\theta_{16}=$	$0.4+0.9\times0.054=0.449$

<u>Question-3</u>:- Apply Self-Organizing Map (SOM) to cluster the A, B, C and D data points for an iteration. Assume that the initial learning rate is 0.8 and the number of clusters to be formed is 2. Also, obtain the learning rate after the first iteration. consider three decimal places for detailed calculation.

Table 1: Data Point for SOM

i	A	B	C	D
1	1	0	0	0
2	0	1	0	1
3	1	1	0	1
4	1	1	1	0

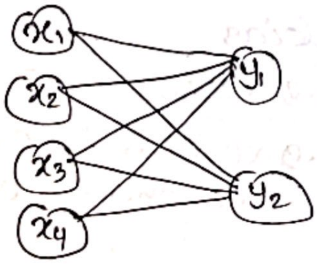
consider the initial weights matrix

$$W = \begin{bmatrix} 0.2 & 0.7 \\ 0.3 & 0.6 \\ 0.4 & 0.4 \\ 0.5 & 0.9 \end{bmatrix}$$

<u>Solution</u>:-

number of inputs = 4

number of clusters = 2



For the First input vector A $[1 \quad 0 \quad 1 \quad 1]$

$$D(j) = \sum_{i=1}^{n} (w_{ij} - x_i)^2$$

$$D(1) = \sum_{i=1}^{4} (w_{i1} - x_i)^2$$

$$D(1) = (0.2-1)^2 + (0.3-0)^2 + (0.4-1)^2 + (0.5-1)^2$$

$$= 1.340$$

and,

$$D(2) = \sum_{i=1}^{4} (w_{i2} - x_i)^2$$

$$= (0.7-1)^2 + (0.6-0)^2 + (0.4-1)^2 + (0.9-1)^2$$

$$= 0.820$$

since $D(1) > D(2)$ therefore wining cluster is $j = 2$, i.e y_2

Update weights on winning cluster units $j = 2$ for input vector A $[1 \quad 0 \quad 1 \quad 1]$

$$w_{ij}(new) = w_{ij}(old) + \alpha [x_i - w_{ij}(old)]$$

$$w_{i2}(new) = w_{i2}(old) + \alpha [x_i - w_{i2}(old)]$$

$$w_{i2}(new) = \begin{bmatrix} 0.7 \\ 0.6 \\ 0.4 \\ 0.9 \end{bmatrix} + 0.8 \begin{bmatrix} 1-0.7 \\ 0-0.6 \\ 1-0.4 \\ 1-0.9 \end{bmatrix} = \begin{bmatrix} 0.940 \\ 0.120 \\ 0.88 \\ 0.98 \end{bmatrix}$$

updated weight matrix, $W = \begin{bmatrix} 0.2 & 0.94 \\ 0.3 & 0.12 \\ 0.4 & 0.88 \\ 0.5 & 0.98 \end{bmatrix}$

For the second input vector B [0 1 1 1]

$\therefore D(1) = (0.2-0)^2 + (0.3-1)^2 + (0.4-1)^2 + (0.5-1)^2$

$= 1.140$

$\therefore D(2) = (0.94-0)^2 + (0.12-1)^2 + (0.88-1)^2 + (0.98-1)^2$

$= 1.673$

$\therefore D(1) < D(2)$, therefore winning cluster is

$j = 1$ i.e. y_1

Update weights on winning cluster units $j = 1$

for input vectors B [0 1 1 1]

$$w_{i1}(new) = \begin{bmatrix} 0.2 \\ 0.3 \\ 0.4 \\ 0.5 \end{bmatrix} + 0.8 \begin{bmatrix} 0-0.2 \\ 1-0.3 \\ 1-0.4 \\ 1-0.5 \end{bmatrix} = \begin{bmatrix} 0.04 \\ 0.86 \\ 0.88 \\ 0.90 \end{bmatrix}$$

$$\text{updated weight matrix, } W = \begin{bmatrix} 0.04 & 0.94 \\ 0.86 & 0.12 \\ 0.88 & 0.88 \\ 0.90 & 0.98 \end{bmatrix}$$

For the third input vector · C $[0 \quad 0 \quad 0 \quad 1]$

$\therefore D(1) = (0.04-0)^2 + (0.86-0)^2 + (0.88-0)^2 + (0.9-1)^2$

$= 1.526$

$D(2) = (0.94-0)^2 + (0.12-0)^2 + (0.88-0)^2 + (0.98-1)^2$

$= 1.673$

$\therefore D(1) < D(2)$; therefore the winning cluster is $j = 1$ i.e. y_1

$$W_{i1}(new) = \begin{bmatrix} 0.04 \\ 0.86 \\ 0.88 \\ 0.9 \end{bmatrix} + 0.8 \begin{bmatrix} 0-0.04 \\ 0-0.86 \\ 0-0.88 \\ 1-0.90 \end{bmatrix}.$$

$$\cancel{0.008} = \begin{bmatrix} 0.008 \\ 0.172 \\ 0.176 \\ 0.98 \end{bmatrix}$$

$$\therefore W = \begin{bmatrix} 0.008 & 0.94 \\ 0.172 & 0.12 \\ 0.176 & 0.88 \\ 0.98 & 0.98 \end{bmatrix}$$

For the fourth input vector D [0 1 1 0]

$$\therefore D(1) = (0.008-0)^2 + (0.172-1)^2 + (0.176-1)^2 + (0.98-0)^2$$

$$= 2.325$$

$$D(2) = (0.94-0)^2 + (0.12-1)^2 + (0.88-1)^2 + (0.98-0)^2$$

$$= 2.633$$

$\therefore D(1) < D(2)$, so, the winning cluster is $j = 1; y_1$

$$W_{i1}(new) = \begin{bmatrix} 0.008 \\ 0.172 \\ 0.176 \\ 0.98 \end{bmatrix} + 0.8 \begin{bmatrix} 0-0.008 \\ 1-0.172 \\ 1-0.176 \\ 0-0.98 \end{bmatrix} = \begin{bmatrix} 0.002 \\ 0.834 \\ 0.835 \\ 0.196 \end{bmatrix}$$

$$\therefore W = \begin{bmatrix} 0.002 & 0.94 \\ 0.834 & 0.12 \\ 0.835 & 0.88 \\ 0.196 & 0.98 \end{bmatrix}$$

cluster	No. of objects	objects
1	3	{B, C, D}
2	1	{A}

Now,

update the $\alpha_{new} = 0.5 \times \alpha_{old} = 0.5 \times 0.8$

$$= 0.4$$

Question: 4 :- consider the distance matrix from a data set and the cluster solution. Compare the Dunn Index for $k=3$ and $k=4$ to find the optimal number of clusters. comment on your results.

$$
\begin{array}{c|ccccc}
 & A & B & C & D & E \\
\hline
A & 0 & & & & \\
B & 60 & 0 & & & \\
C & 69 & 34 & 0 & & \\
D & 194 & 153 & 144 & 0 & \\
E & 24 & 56 & 64 & 188 & 0
\end{array}
$$

K = 3

cluster	no. of Items	Items
cluster 1	2	{A, E}
" 2	2	{B, C}
" 3	1	{D}

K = 4

cluster	no. of items	Items
cluster 1	2	{A, E}
" 2	1	{B}
" 3	1	{C}
" 4	1	{D}

For, $k=3$:,

here, items.

$$
\begin{array}{c|ccccc}
 & A & E & B & C & D \\
\hline
A & 0 & 24 & 60 & 69 & 194 \\
E & 24 & 0 & 56 & 64 & 188 \\
B & 60 & 56 & 0 & 34 & 153 \\
C & 69 & 64 & 34 & 0 & 144 \\
D & 194 & 188 & 153 & 144 & 0
\end{array}
$$

$d_{min} = \min [d_{AB}, d_{AC}, d_{AD}, d_{EB}, d_{EC}, d_{ED}, d_{BD}, d_{CD}]$

$= 56$

$d_{max} = \max [d_{max}(\text{cluster-1}: A, E), d_{max}(\text{cluster-2}: B, C), d_{max}(\text{cluster-3}: D)]$

$= \max [\cancel{8}24, 34, 0]$

$= 34$

$\therefore DI = \dfrac{d_{min}}{d_{max}} = \dfrac{56}{34} = 1.647$

For, $K=4$;

here,

$d_{min} = \min [d_{AB}, d_{EB}, d_{AC}, d_{EC}, d_{AD}, d_{ED}, d_{BC}, d_{BD}, d_{CD}]$

$= 34$

$d_{max} = \max [d_{max}(\text{cluster-1}: A, E), d_{max}(\text{cluster-2}: B), d_{max}(\text{cluster-3}: C), d_{max}(\text{cluster-4}: D)]$

$= \max [24, 0, 0, 0]$

$= 24$

$\therefore DI = \dfrac{d_{min}}{d_{max}} = \dfrac{34}{24} = 1.417$