Response Ratio (RR) $= \frac{\bar{X}_1}{\bar{X}_2}$

$$\ln(RR) = \ln\left(\frac{\bar{X}_1}{\bar{X}_2}\right) = \ln(\bar{X}_1) - \ln(\bar{X}_2)$$

$$V_{\ln(RR)} = \left(\frac{1}{\bar{X}_1}\right)^2 Var(\bar{X}_1) + \left(\frac{1}{\bar{X}_2}\right)^2 Var(\bar{X}_2)$$

$$= \frac{1}{(\bar{X}_1)^2} \frac{\sigma_1^2}{n_1} + \frac{1}{(\bar{X}_2)^2} \frac{\sigma_2^2}{n_2}$$

when, $\sigma_1^2 = \sigma_2^2$

$\sigma^2 = S'^2_{pooled}$

$$\therefore V_{\ln RR} = S^2_{pooled}\left(\frac{1}{n_1 \bar{X}_1^2} + \frac{1}{n_2 \bar{X}_2^2}\right)$$

$$\therefore SE_{\ln(RR)} = \sqrt{V_{\ln(RR)}}$$

95% CI. of $\ln(RR)$

$$\ln RR \pm 1.96\, SE_{\ln(RR)}$$

$LL_{\ln RR} =$ ~~ln RR~~ $\ln RR - 1.96 \times SE_{\ln(RR)}$

$UL_{\ln RR} = \ln RR + 1.96\, SE_{\ln RR}$

$LL_{RR} = \exp(LL_{\ln RR})$

$UL_{RR} = \exp(UL_{\ln RR})$

$$S^2_{pooled} = \frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1+n_2-2}$$

Binary Response variable:

	Event	Non-event	N
Treatment ~~Exposed~~	A	B	n_1
~~NE~~ Control	C	D	n_2

$$RR = \frac{A/n_1}{C/n_2}$$

$$P_1 = \frac{A}{n_1}$$

$$P_2 = \frac{C}{n_2}$$

$$RR = \frac{P_1}{P_2}$$

$$\ln(RR) = \ln(P_1) - \ln(P_2)$$

$$V_{\ln(RR)} = \nabla g^T \Sigma \nabla g$$

$$\frac{\partial g}{\partial P_1} = \frac{1}{P_1}$$

$$\frac{\partial g}{\partial P_2} = \frac{1}{P_2}$$

$$\Sigma = \begin{pmatrix} Var(P_1) & cov(P_1, P_2) \\ cov(P_1, P_2) & Var(P_2) \end{pmatrix}$$

Now,

$$Var(P_1) = Var\left(\frac{A}{n_1}\right)$$

$$= \frac{1}{n_1^2} Var(A)$$

$$= \frac{n_1 P_1(1-P_1)}{n_1^2}$$

$$\therefore Var(P_1) = \frac{P_1(1-P_1)}{n_1}$$

$$Var(P_2) = Var\left(\frac{C}{n_2}\right)$$

$$= \frac{1}{n_2^2} Var(C)$$

$$= \frac{n_2 P_2(1-P_2)}{n_2^2}$$

$$\therefore Var(P_2) = \frac{P_2(1-P_2)}{n_2}$$

$$V_{\ln(RR)} = \nabla g^T \Sigma \nabla g$$

$$\Sigma = \begin{pmatrix} \frac{P_1(1-P_1)}{n_1} & 0 \\ 0 & \frac{P_2(1-P_2)}{n_2} \end{pmatrix}$$

Here,

$$\nabla g^T = \begin{pmatrix} \frac{1}{P_1} & -\frac{1}{P_2} \end{pmatrix}$$

$$\nabla g = \begin{pmatrix} 1/P_1 \\ -1/P_2 \end{pmatrix}$$

$$\therefore V_{\ln(RR)} = \nabla g^T \Sigma \nabla g$$

$$= \begin{pmatrix} \frac{1}{P_1} & -\frac{1}{P_2} \end{pmatrix} \begin{pmatrix} \frac{P_1(1-P_1)}{n_1} & 0 \\ 0 & \frac{P_2(1-P_2)}{n_2} \end{pmatrix} \begin{pmatrix} \frac{1}{P_1} \\ -\frac{1}{P_2} \end{pmatrix}$$

$$= \frac{1}{P_1^2} \times \frac{P_1(1-P_1)}{n_1} + \frac{1}{P_2^2} \frac{P_2(1-P_2)}{n_2}$$

$$= \frac{(1-P_1)}{n_1 P_1} + \frac{(1-P_2)}{n_2 P_2}$$

$$= \frac{1}{n_1 P_1} - \frac{P_1}{n_1 P_1} + \frac{1}{n_2 P_2} - \frac{P_2}{n_2 P_2}$$

We have,

$$P_1 = \frac{A}{n_1}$$

$$\therefore A = n_1 P_1$$

$$P_2 = \frac{C}{n_2}$$

$$\therefore C = n_2 P_2$$

$$\therefore V_{\ln(RR)} = \frac{1}{A} - \frac{1}{n_1} + \frac{1}{C} - \frac{1}{n_2}$$

95% CI for $\ln RR$

$$LL_{\ln RR} = \ln RR - 1.96 \times SE_{\ln RR}$$

$$UL_{\ln RR} = \ln RR + 1.96 \times SE_{\ln RR}$$

95% CI for RR

$$LL_{RR} = \exp(LL_{\ln RR})$$

$$UL_{RR} = \exp(UL_{\ln RR})$$

⊛ Binary response variable:

	E		N
Exposed	A	B	n_1
N·Exposed	C	D	n_2

OR

$$\text{Odds}_1 = A/B$$

$$\text{Odds}_2 = C/D$$

$$OR = \frac{A/B}{C/D} = \frac{AD}{BC}$$

$$\ln(OR) = \ln(A) + \ln(D) - \ln(B) - \ln(C)$$

$$V_{\ln OR} = \nabla g^T \Sigma \nabla g$$

$$\frac{\partial g}{\partial A} = \frac{1}{A} \qquad \frac{\partial g}{\partial C} = -\frac{1}{C}$$

$$\frac{\partial g}{\partial B} = -\frac{1}{B} \qquad \frac{\partial g}{\partial D} = \frac{1}{D}$$

$$\nabla g^T = \begin{pmatrix} 1/A & -1/B & -1/C & 1/D \end{pmatrix}$$

$$\nabla g = \begin{pmatrix} 1/A \\ -1/B \\ -1/C \\ 1/D \end{pmatrix}$$

$$\Sigma = \begin{pmatrix} Var(A) & 0 & 0 & 0 \\ 0 & Var(B) & 0 & 0 \\ 0 & 0 & Var(C) & 0 \\ 0 & 0 & 0 & Var(D) \end{pmatrix}$$

$$P_1 = \frac{A}{n_1} \qquad \therefore \quad A = n_1 P_1$$

$$Var(A) = n_1 P_1 = A$$

$$Var(B) = n_1 (1 - P_1) = B$$

$$Var(C) = n_2 P_2 = C$$

$$Var(D) = n_2 (1 - P_2) = D$$

$$\therefore \Sigma = \begin{pmatrix} A & 0 & 0 & 0 \\ 0 & B & 0 & 0 \\ 0 & 0 & C & 0 \\ 0 & 0 & 0 & D \end{pmatrix}$$

$\therefore V_{\ln OR} = \nabla g^T \Sigma \nabla g$

$$= \left(\frac{1}{A} \quad -\frac{1}{B} \quad -\frac{1}{C} \quad \frac{1}{D}\right) \begin{pmatrix} A & 0 & 0 & 0 \\ 0 & B & 0 & 0 \\ 0 & 0 & C & 0 \\ 0 & 0 & 0 & D \end{pmatrix} \begin{pmatrix} 1/A \\ -1/B \\ -1/C \\ 1/D \end{pmatrix}$$

$$\therefore V_{\ln OR} = \frac{1}{A} + \frac{1}{B} + \frac{1}{C} + \frac{1}{D}$$

$$\therefore SE_{\ln OR} = \sqrt{V_{\ln OR}}$$

95% confidence interval for $\ln OR$

$$LL_{\ln OR} = \ln OR - 1.96 \times SE_{\ln OR}$$

$$UL_{\ln OR} = \ln OR + 1.96 \times SE_{\ln OR}$$

95% CI e for RR

$$LL_{OR} = e^{LL_{\ln OR}}$$

$$UL_{OR} = e^{UL_{\ln OR}}$$

$$\therefore 95\% \text{ CI for } OR = \left[e^{LL_{\ln(OR)}}, e^{UL_{\ln(OR)}}\right]$$

CS CamScanner

RD :

$$RD = \frac{A}{n_1} - \frac{C}{n_2} = P_1 - P_2$$

$$\ln(RD) = \cancel{\ln(P_1) - \ln(P_2)}$$

$$Var(RD) = \frac{1}{n_1^2} Var(A) + \frac{1}{n_2^2} Var(C)$$

$$Var(A) = n_1 P_1 q_1$$

$$\Rightarrow \quad P_1 = \frac{A}{n_1}$$

$$q_1 = \frac{n_1 - A}{n_1} = \frac{B}{n_1}$$

$$Var(A) = \frac{AB}{n_1}$$

$$Var(C) = \frac{CD}{n_2}$$

$$Var(RD) = \frac{1}{n_1^3} AB + \frac{1}{n_2^3} CD$$

$$\therefore \quad SE_{RD} = \sqrt{Var(RD)}$$

95 % CI

$$LL_{RD} = RD - 1.96 \times SE_{RD}$$

$$UL_{RD} = RD + 1.96 \times SE_{RD}$$